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lecture cource on

Astroparticle physics

15.09.2009 – 15.12.2009

3.0 Solar fusion reactions

Basics: Binding energy

The mass of an atom – or nucleus – (^AX) is not a sum of its parts (Z protons and electrons, and N neutrons) but the binding energy is needed to be taken into account

$$m(^AX)c^2 = Z \cdot (m_p + m_e)c^2 + N \cdot m_n c^2 - B$$

The masses are here expressed as atomic mass units u, 1 $u = 931.502$ MeV$/c^2$
The binding energy can be obtained as

$$B = (Z \cdot m(^1H) + N \cdot m_n - m(^AX))c^2$$

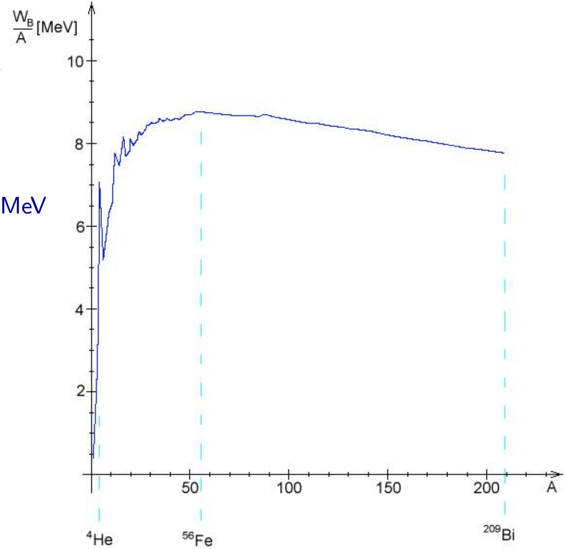
3.0 Solar fusion reactions

Basics: Binding energy systematics

A from 1 to 56
$\Longrightarrow$ fusion reaction
$4\times{}^{1}\mathrm{H} \longrightarrow {}^{4}\mathrm{He}$: $\sim$28 MeV

A from 240 down to 56
$\Longrightarrow$ fission reaction
${}^{238}\mathrm{U} \longrightarrow 2\times{}^{119}\mathrm{Pd}$: $\sim$240 MeV

are releasing energy



3.0 Solar fusion reactions

Basics: Coulomb barrier

An important factor when dealing with fusion reactions is the heigth of the Coulomb barrier

The heigth of their Coulomb barrier, when a and X are the reacting particles, is

$$V_C = \frac{e^2}{4\pi\epsilon_0} \cdot \frac{Z_a \cdot Z_X}{R_a + R_X}, \qquad \frac{e^2}{4\pi\epsilon_0} = 1.44 \text{ MeV} \cdot \text{fm}$$

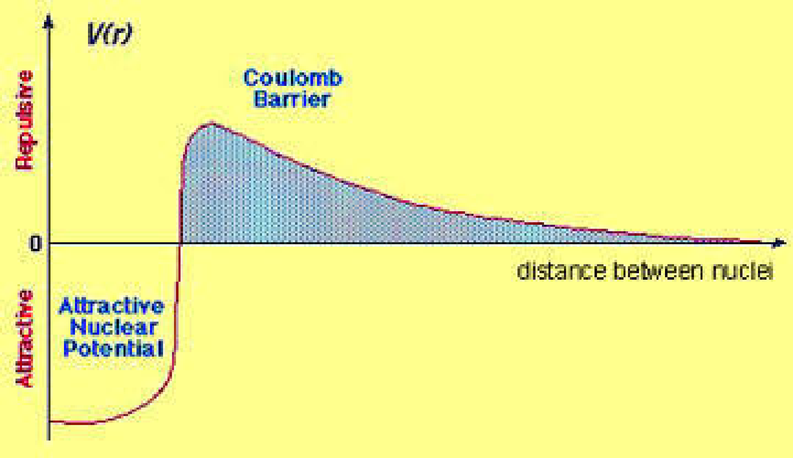
when the particles just touch at their surfaces. Z_a and Z_X are the charges (proton numbers) and R_a and R_X are the radii of the particles

$$R = r_0 \cdot A^{1/3}$$

with $r_0 \approx 1.2$ fm and A is the mass number.

3.0 Solar fusion reactions

Basics: Coulomb barrier



3.0 Solar fusion reactions

Basics: Coulomb barrier

Example:

$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \longrightarrow {}^{2}\mathrm{H} + \mathrm{e}^{+} + \nu$$

Now $Z_a = Z_X = Z_p = 1$, $R_a = R_X = R_p \approx 1.2$ fm, and

$$V_C = \frac{e^2}{4\pi\epsilon_0} \cdot \frac{Z_a \cdot Z_X}{R_a + R_X} = 1.44 \, \frac{Z_p^2}{2R_p} \text{ MeV} \cdot \text{fm} = 0.6 \text{ MeV}$$

With strong force included, the effective barrier, for the present reaction, is

$$V_C = 0.550 \text{ MeV}$$

In classic terms, this $p + p$ -reaction occur only when the energy of the protons exceeds 0.550 MeV (*)

3.0 Solar fusion reactions

Basics: Nuclear reaction Q-value

The nuclear reaction (and decay) can be characterised by its Q-value

$$Q = (M_{\text{initial}} - M_{\text{final}})c^2 = (m_a + m_X - m_b - m_Y)c^2$$

- $Q > 0$: exothermic
  nuclear mass or binding energy is released as kinetic energy of the final products
- $Q < 0$: endothermic
  the initial kinetic energy is converted into nuclear mass or binding energy

Example: $^1\text{H} + {}^1\text{H} \longrightarrow {}^2\text{H} + \ldots \quad (p + p \longrightarrow d)$
$m(^1\text{H}) = 1.007825\text{u}$
$m(^2\text{H}) = 2.014102\text{u}$
$Q = (m_p + m_p - m_d)c^2 = 1.44$ MeV
This energy goes to the kinetic energy of positron and neutrino.

3.1 Solar nuclear reactions

Main processes

Temperature [10^6K]	Process – burning	Reaction product	Mass of a star [M_{SUN}]
10–20	hydrogen pp-chain	helium	$\sim$1
	CNO cycle	helium	$\sim$1.5
100–200	helium triple-α	carbon, oxygen	
500	carbon	Ne, Na, Mg (16$\leq$A$\leq$28)	$\sim$3
1000	oxygen	Si, S, P (16$\leq$A$\leq$28)	
2000–4000	silixon	Fe, Ni (28$\leq$A$\leq$56)	$\sim$15
	s-, r-, and p-processes	A$\geq$60	

3.1 Solar nuclear reactions

Reaction rates – 1

Stellar reaction rate $\langle \sigma v \rangle$ can be written using Maxwell-Boltzmann energy distribution (in the centre-of-mass system, for a reaction pair)

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{1/2} \cdot \frac{1}{kT^{3/2}} \cdot S(E_0) \int_0^\infty \exp\left(-\frac{E}{kT} - \frac{b}{E^{1/2}}\right) dE$$

where $k = 8.6171 \times 10^{-5}$ eV$\cdot$K^{-1} is the Boltzmann constant, T the temperature, and μ the reduced mass.

- S-factor contains information about the cross section. It is nearly constant at solar energies; $S(E) \sim 100 - 1000$ MeV$\cdot$b
- Term $b/E^{1/2}$ describes the penetration through the Coulomb barrier;
  $b = (2\mu/\hbar^2)^{1/2} \pi e^2 \cdot Z_1 \cdot Z_2 = 0.989 \cdot Z_1 \cdot Z_2 \cdot \mu^{1/2}$ (MeV)$^{1/2}$
  b^2 is also called Gamow energy (E_G)

3.1 Solar nuclear reactions

Reaction rates – 2

- The effective mean energy for thermonuclear fusion reaction (at given temperature T) can be obtained as

$$E_0 = \left(\frac{bkT}{2}\right)^{2/3}$$

For $^1H + {}^1H$ (at $T \sim 15 \times 10^6$ K) : $E_0 = 6$ keV (**)

- Comparing (*) and (**): $V_C = 550$ keV vs. $E_0 = 6$ keV $\Longrightarrow$ the fusion of $^1H + {}^1H$ occurs via tunneling through the Coulomb barrier
  $\Longrightarrow$ the rate is very slow, for this reaction $\sim$1 per 10^9 years, which keep the sun shining billions of years
- About 600 million tons of hydrogen is burned into helium per second in the sun.

3.1 Solar nuclear reactions

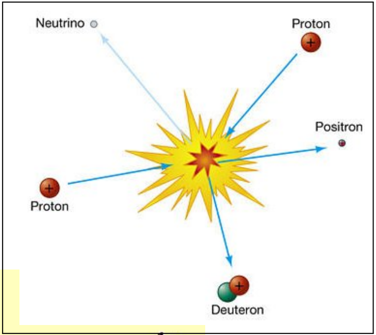
Energy production

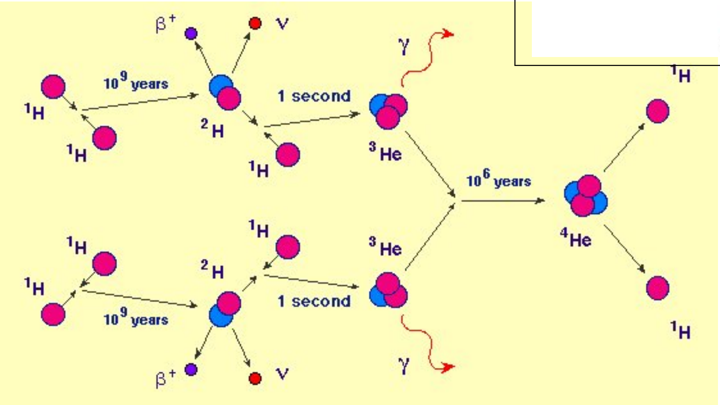
The (main) energy production in stars is competition between two options

- Proton-proton chain
  - the most important mechanism below $\sim$1.5 M_{SUN} ($T_6 < 20$)
  - includes three branches, but the first (ppI) dominating
  - net result: converts 4 protons into a helium
- CNO cycle
  - carbon-nitrogen-oxygen cycle
  - dominant process above $\sim$1.5 M_{SUN}, but in the sun produces $\sim$2%
  - large astronomical relevance
  - net result: converts 4 protons into a helium
    $\longrightarrow$ carbon, nitrogen and oxygen serve as catalysts

3.1 Solar nuclear reactions

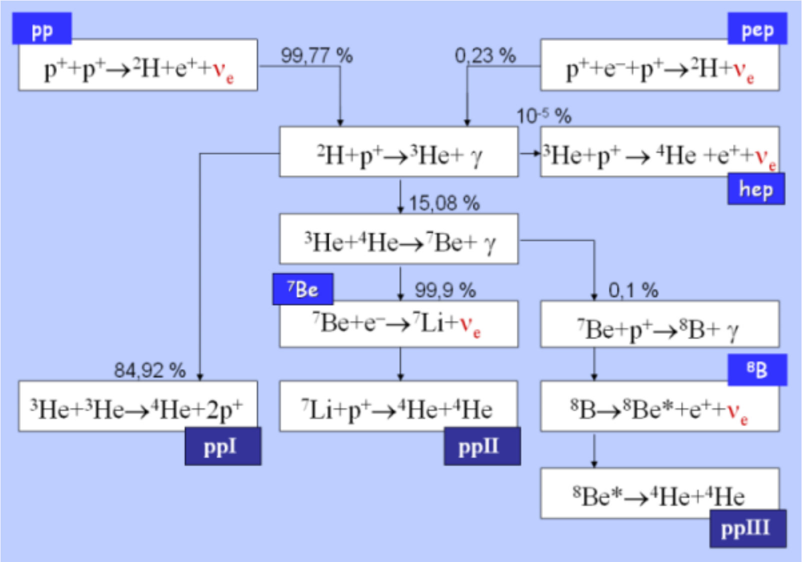
Energy production: proton–proton chain: ppI





3.1 Solar nuclear reactions

Energy production: proton–proton chain: all



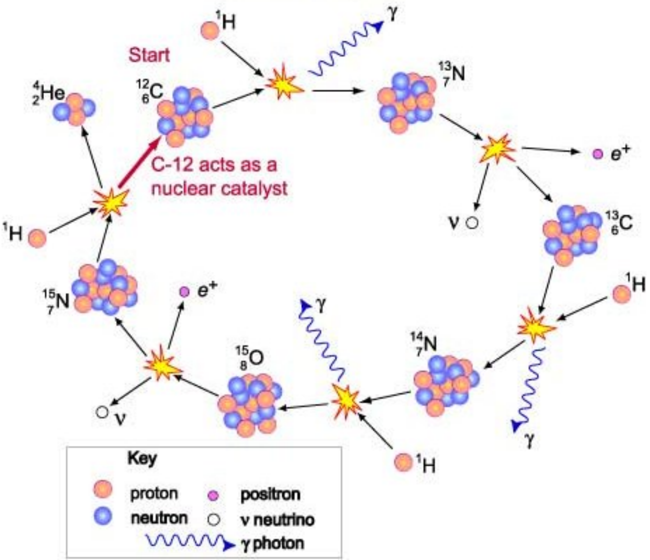
3.1 Solar nuclear reactions

Energy production: CNO cycle – 1

- The CNO cycle consists of the following reactions:
  (1) $^{12}C + {}^{1}H \longrightarrow {}^{13}N + \gamma$
  (2) $^{13}N \longrightarrow {}^{13}C + e^{+} + \nu$ $\qquad (E_\nu \leq 1.2$ MeV$)$
  (3) $^{13}C + {}^{1}H \longrightarrow {}^{14}N + \gamma$
  (4) $^{14}N + {}^{1}H \longrightarrow {}^{15}O + \gamma$
  (5) $^{15}O \longrightarrow {}^{15}N + e^{+} + \nu$ $\qquad (E_\nu \leq 1.73$ MeV$)$
  (6) $^{15}N + {}^{1}H \longrightarrow {}^{12}C + {}^{4}He$
- The CNO is the dominant process above $\sim$1.5 M_{SUN}
- In the sun it contributes $\sim$2%
- Large astronomical relevance (age of globular clusters)
- The "bottle-neck" reaction $^{14}N + {}^{1}H \longrightarrow {}^{15}O + \gamma$ (4) is the slowest and determines the rate of the cycle
  - at $T_6 = 20$, about 1 reaction/million years
  - the newest LUNA results: rate slower than expected by solar models
  - solar CNO–ν prediction lowered by factor of 2

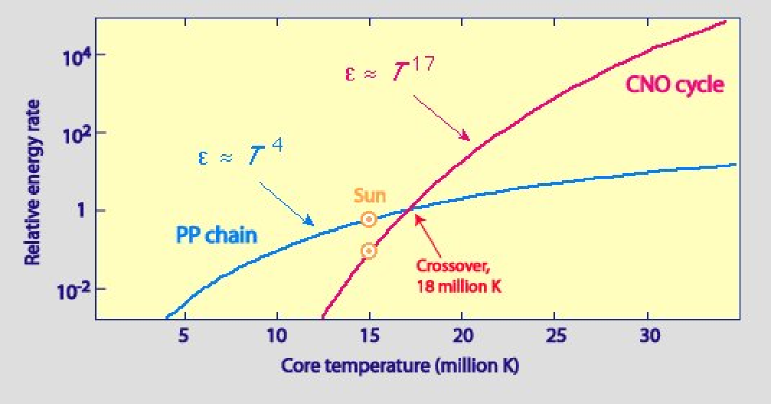
3.1 Solar nuclear reactions

Energy production: CNO cycle – 2



3.1 Solar nuclear reactions

Energy production: reaction rates of pp vs. CNO



3.1 Solar nuclear reactions

Helium burning: triple-α and α-reactions

- Amount of helium increaces due to pp-chain and CNO-cycle
- At approximately $T = 10^8$ K, triple-α reaction starts to burn helium into carbon
- The triple-α reactions are
  (1) $^4\text{He} + {}^4\text{He} \longleftrightarrow {}^8\text{Be}$
  (2) $^8\text{Be} + {}^4\text{He} \longrightarrow {}^{12}\text{C} + \gamma$
  ^{8}Be is highly unstable isotope ($\tau \approx 30$ fs) and decays back to two α-particles
  $\Longrightarrow$ resonance in ^{12}C
- Helium may fuse also with carbon and heaver nuclei
  (i) $^{12}\text{C} + {}^4\text{He} \longrightarrow {}^{16}\text{O} + \gamma$
  (ii) $^{16}\text{O} + {}^4\text{He} \longrightarrow {}^{20}\text{Ne} + \gamma$
  (iii) $^{20}\text{Ne} + {}^4\text{He} \longrightarrow {}^{24}\text{Mg} + \gamma$
- These reactions, however, are not very important in the energy production

3.1 Solar nuclear reactions

Carbon, oxygen and silicon burning

- After helium has been exhausted, carbon burning starts at temperature $T = 5 - 8 \times 10^8$ K

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\longrightarrow {}^{24}\mathrm{Mg} + \gamma \longrightarrow {}^{23}\mathrm{Mg} + \mathrm{n} \\
&\longrightarrow {}^{23}\mathrm{Na} + {}^{1}\mathrm{H} \\
&\longrightarrow {}^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \\
&\longrightarrow {}^{16}\mathrm{O} + 2\,{}^{4}\mathrm{He}
\end{aligned}
$$

- Oxygen burning take place in a slightly higher temperature

$$
\begin{aligned}
{}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\longrightarrow {}^{32}\mathrm{S} + \gamma \longrightarrow {}^{31}\mathrm{S} + \mathrm{n} \\
&\longrightarrow {}^{31}\mathrm{P} + \mathrm{n} \\
&\longrightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He} \\
&\longrightarrow {}^{24}\mathrm{Mg} + 2\,{}^{4}\mathrm{He}
\end{aligned}
$$

- The reactions stop in silicon burning

$$
\begin{aligned}
{}^{28}\mathrm{Si} + {}^{28}\mathrm{Si} &\longrightarrow {}^{56}\mathrm{Ni} + \gamma \\
{}^{56}\mathrm{Ni} &\longrightarrow {}^{56}\mathrm{Fe} + 2e^{+} + 2\nu_e
\end{aligned}
$$

- These and the following phenomena are the topic of the next lecture. Now we continue with neutrinos

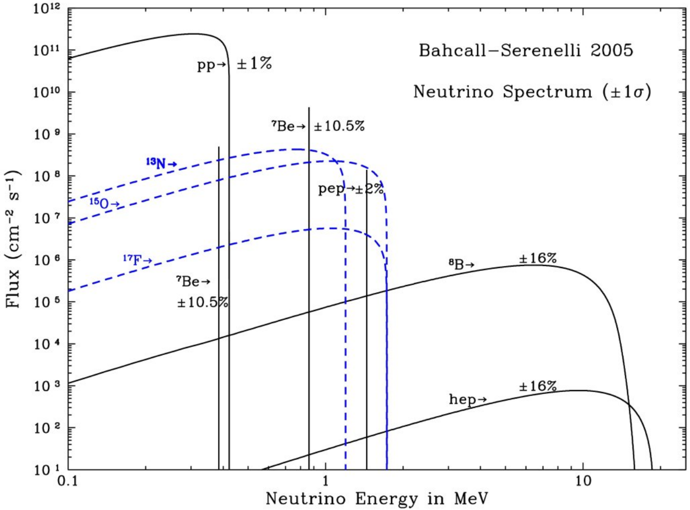
3.2 Solar neutrinos

Reactions producing neutrino: pp and CNO

- pp: $^1\mathrm{H} + {}^1\mathrm{H} \longrightarrow {}^2\mathrm{H} + e^+ + \nu_e$
- ^{7}Be: $^7\mathrm{Be} + e^- \longrightarrow {}^7\mathrm{Li} + \nu_e$
- ^{8}B: $^8\mathrm{B} \longrightarrow {}^8\mathrm{Be}^* + e^+ + \nu_e$
- pep: $p + e + p \longrightarrow {}^2\mathrm{H} + \nu_e$
- hep: $^3\mathrm{He} + p \longrightarrow {}^4\mathrm{He} + e^+ + \nu_e$
- CNO: $^{13}\mathrm{N} \longrightarrow {}^{13}\mathrm{C} + e^+ + \nu_e$
- CNO: $^{15}\mathrm{O} \longrightarrow {}^{15}\mathrm{N} + e^+ + \nu_e$

3.2 Solar neutrinos

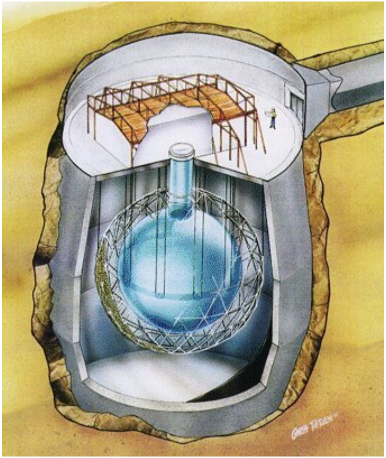
Neutrino spectrum



3.2 Solar neutrinos

SNO – the detector

- Deep underground (6800 ft, 2070 m, 5900 mwe) in the Creighton Mine (owned by INCO Ltd) near Sudbury, Canada (Ontario)
- Water Cherenkov detector, 1 kton of D_2O (12-m diameter acrylic vessel) and 5 kton of H_2O as shielding
- Over 9000 PMT's
- Energy threshold $\sim$5 MeV
- Number of deuteron atoms $\sim 6 \times 10^{31}$
- Neutrino Sensitivity: ^{8}Be and hep
- Operated (with heavy water): 1999–2002



3.2 Solar neutrinos

SNO – detection of neutrinos – 1

- Heavy water (D_2O) allows several reactions to be used to detect neutrinos
  - (1) $\nu_e + d \longrightarrow e^- + p + p, \qquad E_{thr} = 1.442$ MeV (charged current, CC)
  - (2) $\nu + e^- \longrightarrow \nu + e^-$ (elastic scattering, ES)
  - (3) $\nu + d \longrightarrow \nu + p + n, \qquad E_{thr} = 2.225$ MeV (neutral current, NC)

    $n + d \longrightarrow {}^3He + \gamma(6.3 \text{ MeV})$

  ☞ SNO can measure all neutrino flavors (using (3))

- To enhance the detection efficiency of neutrons in reaction (3), Cl – in the form of 2 tons of NaCl – has been, for example, added to the heavy water
  - $n + {}^{35}Cl \longrightarrow {}^{36}Cl + \gamma(8.6 \text{ MeV})$

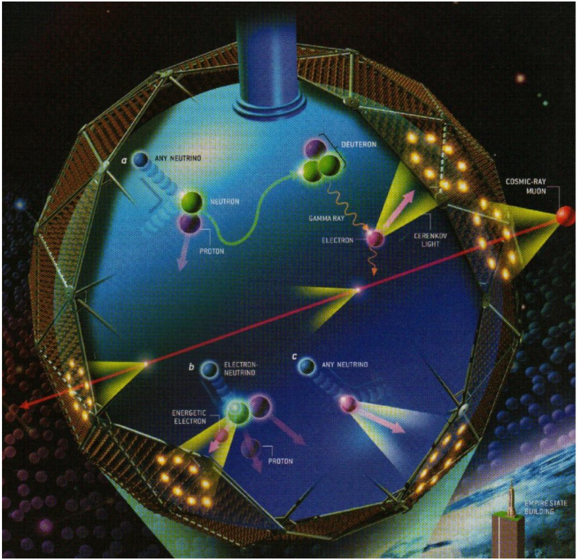
3.2 Solar neutrinos

SNO – detection of neutrinos – 2

(c) $\nu + e^- \longrightarrow \nu + e^-$
(ES)

(b) $\nu_e + \mathrm{d} \longrightarrow e^- + \mathrm{p} + \mathrm{p}$
(CC)

(a) $\nu + \mathrm{d} \longrightarrow \nu + \mathrm{p} + \mathrm{n}$,
(NC)
$\mathrm{n} + \mathrm{d} \longrightarrow {}^3\mathrm{He} + \gamma(6.3\ \mathrm{MeV})$



3.2 Solar neutrinos

SNO – ^{8}B neutrinos

- measurement time : 306 days (Nov. 1999 – May 2001)
- Solar neutrino events (D_2O)
  - CC events : $1967.7\ ^{+61.9}_{-60.9}$
  - NC events : $576.5\ ^{+49.5}_{-48.9}$
  - ES events : $263.6\ ^{+26.4}_{-25.6}$
- Measured ^{8}B fluxes
  - $\Phi_e = 1.76 \pm 0.05(\text{stat}) \pm 0.09(\text{syst}) \times 10^6 \quad \text{cm}^{-2} \cdot \text{s}^{-1}$
  - $\Phi_{\mu\tau} = 3.41 \pm 0.45(\text{stat})\ ^{+0.48}_{-0.45}(\text{syst}) \times 10^6 \quad \text{cm}^{-2} \cdot \text{s}^{-1}$
- References

  - Q.R. Ahmad *et al.*, (SNO-Collaboration), PRL**87**(2001)071301-1,
  - Q.R. Ahmad *et al.*, (SNO-Collaboration), PRL**89**(2002)011301-2

3.2 Solar neutrinos

Borexino – the detector – 1

- Liquid scintillation detector at the Gran Sasso laboratory, Italy, at the depth of 3800 mwe
  - muon flux suppressed by $\sim 10^6$
- Scintillator: 300 tons of pseudocumene
- Outer diameter (water tank) 18 metres, inner diameter (scintillator) 13.7 m
- PMT's: 2212 (8") for inner volume and 208 (8") for outer volume
- The main goal is to measure solar neutrinos of ^{7}Be (electron capture, 862 keV)
  - scintillation detector for low-energy neutrinos: good light yield (compare with water Cherenkov), but no direction information
- The energy threshold $\sim$200 keV
  - set mainly by contamination from ^{14}C
- Operated in full scale $\sim$1 year

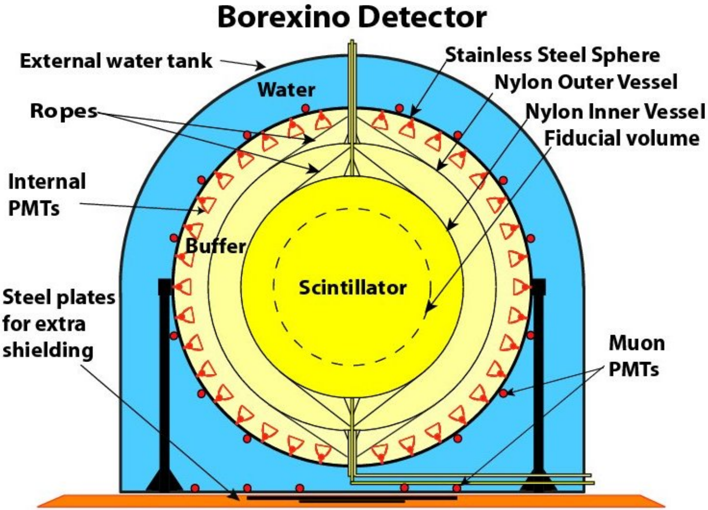
3.2 Solar neutrinos

Borexino – the detector – 2

- Enormous amount of work has been carried out to obtain Borexino as ultra-pure experiment
  - ultra-pure: radioactive contamination as low as possible
  - detector materials need to be several order of magnitude cleaner (less radioactive) than, for example, drinking water
- The main requirement
  - radioactive contamination below the expected interaction rate of 0.5 counts/(day·ton) for ^{7}Be
- The observed radioactive background much lower than expected
  - also CNO and pep, and possibly pp and ^{8}B
  - also geoneutrinos and supernova neutrinos
- All neutrino flavours are detected by ES, and for electron anti-$\nu's$ also by inverse β-decay (on protons and carbon)
  - the recoil energy of electrons or positrons is converted into scintillation light

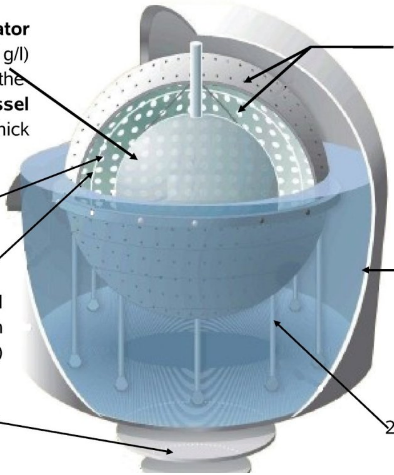
3.2 Solar neutrinos

Borexino – the detector – 3



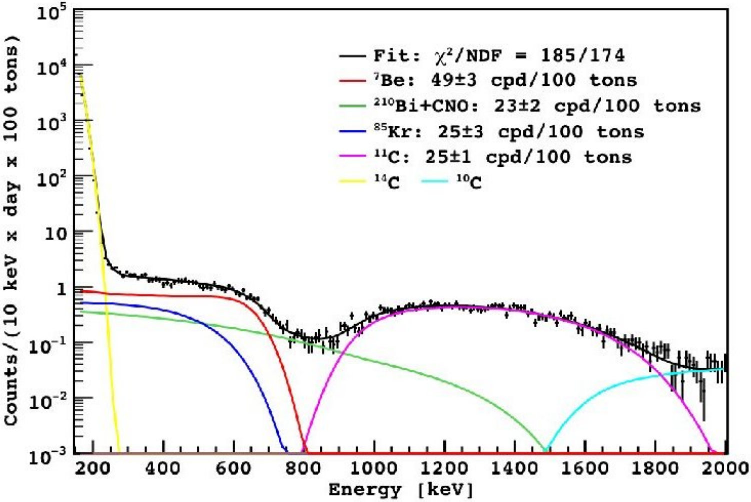
3.2 Solar neutrinos

Borexino – the detector – 4



3.2 Solar neutrinos

Borexino – preliminary results for ^{7}Be



3.2 Solar neutrinos

Borexino – first results and future

- Scattering rate of ^{7}Be solar-ν on electrons $47 \pm 7_{\mathrm{STAT}} \pm 12_{\mathrm{SYS}}$ counts/day/100 tons [astro-ph 0708.2251v2]

- Measure ^{7}Be neutrino flux with high precision ($< 10\%$)
- Extend ^{8}B neutrino spectrum to low energies – spectrum shape gives information for oscillation
- Measure flux of CNO neutrinos

- Prototype for LENA (Low Energy Neutrino Astronomy) which is one of the LAGUNA detectors