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1. The Sun converts protons into helium according to the reaction (ppI)

$$4p \longrightarrow {}^4\mathrm{He} + 2e^+ + 2\nu_e.$$

The solar constant describing the power of the Sun at the Earth is $P \approx 1400$ W$\cdot$m^{-2}. The energy gain of the ppI-reaction corresponds to the binding energy of helium, $E_\mathsf{B}({}^4\mathrm{He}) \approx 28$ MeV.

How many solar ppI-neutrinos arrive at Earth?

2. Estimate the time required for a photon – created, for example, in the ppI-chain – to "travel" from the centre of the Sun to its surface.

Hints. You may use
– *random walk* approximation to describe the travel of the photon,
– equation $\lambda = 1/\kappa\rho$ to relate the mean free path (λ) of a photon in the Sun, the mass absorption coefficient (κ) and the density (ρ) of the Sun.
At $p \approx 10^3$ kg$\cdot$m^{-3} and $T \approx 10^6$ K, $\kappa \approx 10^2$ m$^2\cdot$kg^{-1}.

3. (a) Using the relations

$$v = \frac{p}{E} \quad \text{and} \quad E^2 = p^2 + m^2 (c = 1)$$

for the velocity and energy of a relativistic neutrino, where p and m are the momentum and mass of the neutrino, respectively, derive the equation

$$\Delta t = 0.0515 \left(\frac{mc^2}{E}\right)^2 \cdot L[\mathrm{kpc}]$$

for the time-of-flight delay of massive neutrinos (from supernova explosions).
Hint. For $m \ll E$, $(1 - x)^{1/2} = 1 - \frac{1}{2}x$ (x small).

(b) Estimate, using the above-derived equation, the upper limit for the neutrino mass from the neutrino bursts observed by Kamiokande II and IMB detectors. The distance to the SN1987A is 51.8 kpc.

4. Supernova neutrino spectrum can be expressed as

$$f_\nu(E_\nu) = \frac{1}{T_\nu^3 F_2(0)} \frac{E_\nu^2}{\exp\{\mathrm{E}_\nu/\mathrm{T}_\nu\} + 1}$$

with $T_\nu = 4$, 5 and 8 MeV for ν_e, $\bar{\nu}_e$, ν_x $(x = \mu, \tau)$, respectively.

Assume a spherical detector of the radius of r = 3 metres and filled with xenon gas at the pressure of P = 10 bar. The interaction process being the coherent neutrino nucleus scattering in the xenon gas, the cross section can be expressed as

$$\sigma = C \cdot N_{\mathrm{n}}^2 \cdot E_\nu^2,$$

with xenon obeying the ideal-gas law (C = 4.242×10^{-45} cm^2 MeV^{-2}, N_{n}=76).

The total neutrino energy release can be taken as $W_\nu = 2 \times 10^{59}$ MeV, equally partitioned to all six flavours.

How many neutrino events such a detector would observe from stardard supernova?
No oscillations assumed.

Hints.
Luminosity at the Earth is

$$L_\nu = \frac{1}{4\pi D^2} \frac{W_\nu}{< E_\nu >},$$

where $<\mathrm{E}_\nu> = \mathrm{T}_\nu\ \mathrm{F}_3(0)/\mathrm{F}_2(0)$ is the mean neutrino energy, and

$$F_3(0) = \int \frac{x^3\ dx}{exp(x) + 1} = 5.682.$$